Quantum Wave Mechanical Model
Part One

Bohr's model of the atom was both a success and a failure. It successfully predicted the frequencies of the lines in the hydrogen spectrum and it adequately explained how atomic spectra worked.

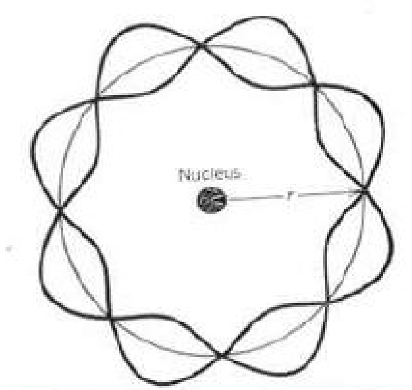
There were several problems that bothered physicists and chemists:

- The model was a total failure when it tried to predict energy levels for atoms with more than one electron.
- Why should electrons be confined to only specified energy levels?
- Why don't electrons give off light all of the time?
  - As electrons change direction in their circular orbits (i.e., accelerate), they should give off light.
- The Bohr model could explain the spectra of atoms with one electron in the outer shell very well, but was not very good for those with more than one electron in the outer shell.
- Why could only two electrons fit in the first shell and why eight electrons in each shell after that? What was so special about two and eight?

Obviously, the Bohr model was missing something!

In 1924, a French physicist named **Louis de Broglie** suggested that, like light, electrons could act as both particles and waves. De Broglie's hypothesis was soon confirmed in experiments that showed electron beams could be diffracted or bent as they passed through a slit much like light could. So, the waves produced by an electron confined in its orbit about the nucleus sets up a standing wave of specific wavelength, energy and frequency (i.e., Bohr's energy levels) much like a guitar string sets up a standing wave when plucked.

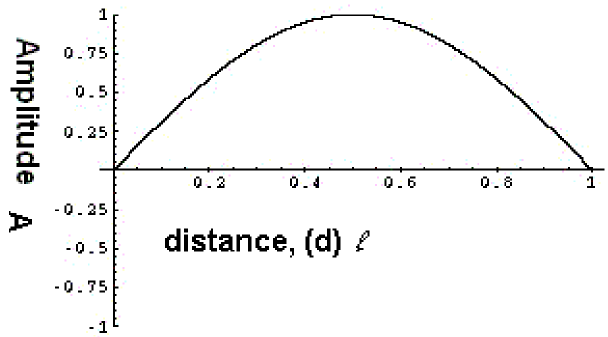
De Broglie's vision of Bohr's atom



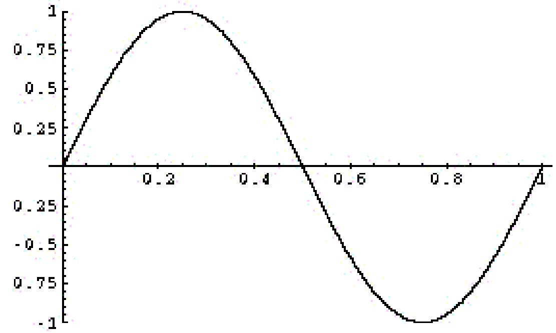
The diagram above illustrates an electron standing wave vibrating in an orbit around a nucleus of an atom. Only integral numbers of wavelengths are allowed. Below is a series of diagrams that illustrate the allowed vibrations of a string fixed on both ends. If a string is fixed on both ends, then the only waves that can occur are those with zero amplitude at those fixed ends; such points of zero amplitude are called *nodes*. Below we show four of the infinite number of vibrations with a node at each end. These vibrations are called **standing waves**.

One String Vibrating - 1/2 wavelength - One loop, two nodes quantum number n = 1 - Click on Graphic for animation

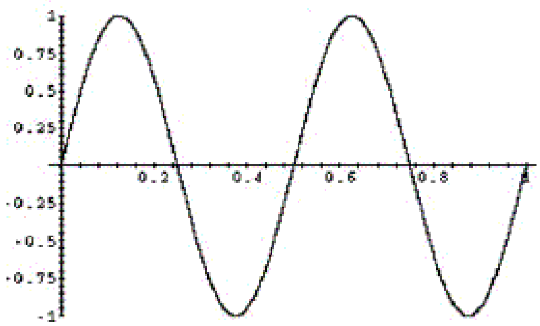
One String Vibrating - 1 wavelength - 2 loops, 3 nodes



quantum number n = 2 - Click on Graphic for Animation



One String Vibrating - 2 wavelengths - 4 loops, 5 nodes
quantum number n = 4 - Click on Graphic for Animation



De Broglie carried the idea of standing waves to the Bohr atom. Standing waves in a circular orbit can exist only if the circumference of the orbit is an integral number of the wavelengths (see figure 1). For a standing wave around the orbit the following must be true

$2\pi r = n\lambda$

The electrons moving in an orbit have a certain momentum given by the expression

$p = mv$

The wavelength of the electron can be expressed as a function of momentum

$p = h/\lambda$

Substituting for momentum De Broglie got

$mv = h/\lambda$

Solving for λ de Broglie derived the following relationship

$\lambda = h/mv$

where h is Planck's constant, m is the mass of the particle, and v is the velocity.

De Broglie proposed this relationship as a general one. With every particle, there is an associated wave. The wavelength of the particle depends on its mass and how fast it is moving.

Sample problems:

1. What is the de Broglie wavelength of a person with a mass of 50 kg jogging at 5 m/s ?

   Solution:

   Since $\lambda = h/mv$ substitute the values for the know variables and solve for λ.

$\lambda = 6.626 \times 10^{-34}$ Js / (50 kg)(5 m/s)
$= 2.65 \times 10^{-36}$ m

For a large particle like the jogger there is NO WAY of seeing this wavelength, but we can apply the same principle to smaller particles such as electrons.

2. What is the de Broglie wavelength of an electron moving at 2.2×10^6 m/s?

Mass of electron is 9.1×10^{-31} kg

$\lambda = h / mv$

$\lambda = 6.626 \times 10^{-34}$ Js/ $(9.1 \times 10^{-31}$ kg$)(2.2 \times 10^6$ m/s)
= 0.332 nm

This is a scale at which noticeable effects such as diffraction patterns, and Doppler effects can be observed

3. Electrons experience a drop in energy of 6.409×10^{-15} J, what is the wavelength of the electrons?

To calculate the velocity of the electrons the expression $E = \frac{1}{2} m_e v^2$ is used.

$$v = \sqrt{\frac{2E}{m_e}} = \sqrt{\frac{2 \times 6.409 \times 10^{-15} kgm^2 s^{-2}}{9.110 \times 10^{-31} kg}} = \sqrt{1.407 \times 10^{16} m^2 s^{-2}} = 1.186 \times 10^8 ms^{-1}$$

Using de Broglie's equation calculate λ

$$\lambda = \frac{h}{m_e v} = \frac{6.6262 \times 10^{-34} Js}{9.110 \times 10^{-31} kg \times 1.186 \times 10^8 ms^{-1}} = 0.06135 \times 10^{-10} \frac{kgm^2 s^{-2} s}{kgms^{-1}} = 6.135 \times 10^{-12} m$$

Problems:

1. Find the de Broglie wavelength of :
a) a 55 g rock flying through the air with a velocity of 35 m/s.

b) an 800 keV electron. **(The conversion from eV, electron volts, to J, Joules is 1eV = 1.6022×10^{-19} J)**

Answers: a) 3.44×10^{-34} m, b) 2.02×10^{-12} m

2. What is the de Broglie wavelength of each of the following objects? (a) A 55-kg block dropped off of a 25-m building just before it hits the 2ground. (b) You in your car driving down the road at 60 mph. (Assume that your car with you in it has a mass of about 1500 kg.)

Answers: a) 5.4×10^{-37} m, b) 1.65×10^{-38} m.

3. What is the de Broglie wavelength of the following particle? An electron accelerated to a total energy of **700 keV**.

Answer: 2.6×10^{-12} m

4. a) How fast must a 200 g baseball travel to have the same de Broglie wavelength as a 40kV electron?
b) How long would it take the baseball to travel the distance of a carbon – carbon bond, with a bond length of 1.54 Å ?

Answers: a) 0.540×10^{-21} m s^{-1}, b) >10 000 years